Maths Week/Wiki Pāngarau 2025

Maths Extra!/Pāngarau taapiri!

Working with collections of things

For students

WORKING WITH COLLECTIONS OF THINGS

In maths, a **set** is a collection of items which have put together, usually for some particular purpose. For example, we could have the set of whole numbers from 1 to 20, the set of letters of the alphabet or the set of members of a sports team. A set is shown by curly brackets { }, and is commonly named by a capital letter.

Each member of a set is an **element** of the set – as an example, the set of vowels V = {a, e, i, o, u} has five elements. The number of elements in a set is the **cardinal number** of the set; the cardinal number of the set V is 5, and we write this as n(V) = 5.

Question 1

List Q = {quadrilaterals}. (There are seven different quadrilaterals.)

Words that we use with sets

The **universal** set for a particular situation is the set of all elements relevant to that situation. We use the symbol $\mathcal{U}$ for a universal set.

The **null** set is the set that has no elements. We write the null set as { } or using the symbol ϕ (the Greek letter phi).

A **subset** of a set A is a set of elements all of which are in A.

The **intersection** of a number of sets is the set of elements that are in **all** the sets; we use the symbol ∩ to show intersection. The key idea for intersection is <u>and</u> – to indicate elements in <u>both</u> or <u>all</u> sets.

The **union** of sets is the set of elements that is in **any** (or all) of the sets; we use the symbol ∪ to show intersection. The symbols for universal set and union are very similar - be careful not to confuse them. The key idea for union is <u>or</u>, which could mean an element is in one set or both sets.

The **complement** of a set S is the set of elements that are in the universal set but not in S. We use the symbol S′ for the complement of set S, and read it as "S complement". The key idea for complement is <u>not</u>.

For example, if

$\mathcal{U}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {1, 2, 3, 4, 5} and

B = {3, 5, 7, 9} then

- A ∩ B = {3, 5}
- A ∪ B = {1, 2, 3, 4, 5, 7, 9}
- A′ = {6, 7, 8, 9, 10}.

Check each number carefully to see why is has been listed where it has been.

Note that although 3 and 5 are in both A and B, we don't repeat them in the listing of A ∩ B or A ∪ B.

Question 2

Let $\mathcal{U}$ = {red, orange, yellow, green, blue, indigo, violet}, P = {red, yellow, blue}, Q = {indigo violet}, R = {red, yellow, orange} and S = {orange, yellow, green}.

(a) List P ∪ Q.

(b) List R ∩ S.

(c) List P ∩ Q.

(c) List Q′.

(d) What is n(P)?

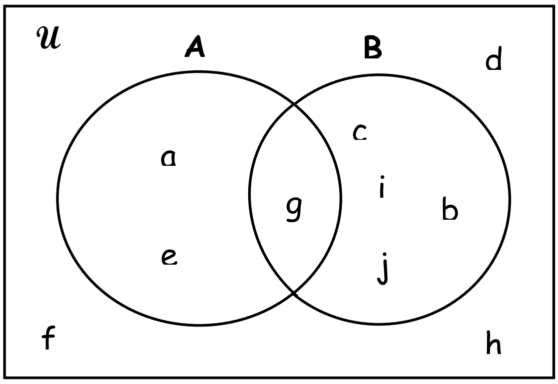
(e) What is n(P ∪ Q)?

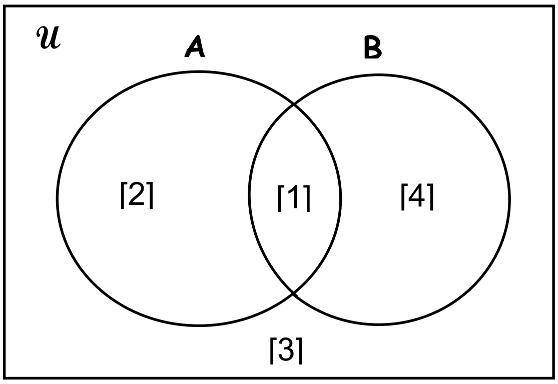
(f) List P ∩ R ∩ S.

Venn Diagrams

Drawing a diagram of sets can be very helpful when working with sets. Such diagrams are called **Venn diagrams** because they were developed by the English logician John Venn. In a Venn diagram, the universal set is drawn as a rectangle, and other sets that are subsets of the universal set are shown in circles inside the rectangle.

The diagrams below are Venn diagrams. In the diagram on the left, there are two subsets of $\mathcal{U}$, A and B, and the elements are shown in various parts of the diagram. In the diagram on the right, the number of elements in the various parts of the Venn diagram on the left are shown – the numbers are put in brackets to show that they are not elements. Study the diagrams carefully so that you understand what the various parts of them show.





Venn diagrams are very useful in problems of logic and in probability.

Question 3

Use the Venn diagram above to answer the following questions.

(a) List A.

(b) List A ∩ B.

(c) List A ∪ B.

(d) List A′.

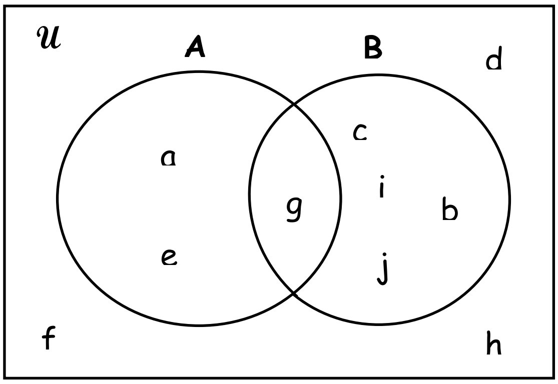
Question 4

(a) If $\mathcal{U}$ = {1, 2, 3, 4, 5, 6, 7, 8}, E = {even numbers} and P = {prime numbers}, draw a Venn diagram to show this.

(b) If $\mathcal{U}$ = {quadrilaterals} (as in question 1), E = {quadrilaterals with equal length sides}, and L = {quadrilaterals with at least one pair of parallel sides}, draw a Venn diagram to show this.

Question 5

Here's the Venn diagram from the previous page again. Use it to answer the following questions.



(a) Which elements are in A and not also in B? Write this as a set and name it using A and B and other set symbols.

(b) Which elements are in neither A nor B? Write this as a set, and name it using A and B and other set symbols.

(c) List $(A \cap B)'$.

(d) List $A' \cap B$.

(e) What is $n(A)$?

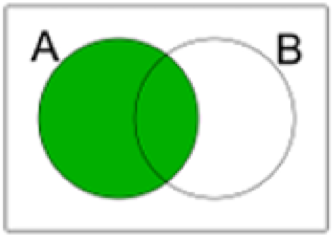
(f) What is $n(\mathcal{U})$?

(g) What is $n(A \cup B)$?

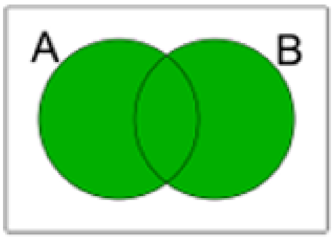
(h) What is $n(A \cap B')$?

Question 6

Shading is often used on Venn diagrams, as in the examples below (the region shaded is named under each diagram).



A



A ∪ B

Draw diagrams to show the following for two sets A and B. Hint: the key ideas "and", "or" and "not" should be helpful.

(a) (i) $A \cap B$
(ii) A'
(iii) $A \cap B'$
(iv) $(A \cup B)'$
(v) $A' \cap B'$
(vi) $A' \cup B$

(b) Draw a Venn diagram of two sets P and Q for which $P \cap Q = \phi$.

(c) Draw a Venn diagram of two sets X and Y for which $X \cup Y = X$.

Question 7

In a group of 30 students, 15 take Geography, 12 take History, and 9 take both subjects. Draw a Venn diagram to show this. Use appropriate symbols for the subsets in your Venn diagram.

How many take neither subject?

Question 8

In the same group of 30 in question 7:

- 1 plays cricket and football but not tennis
- 2 play tennis and football but not cricket
- 7 play cricket and tennis but not football
- 4 play football but not cricket or tennis
- 3 play tennis but not cricket or football
- 6 play cricket but not football or tennis
- 2 play all three sports.

(a) How many play none of the three sports?

(b) How many play cricket?