



Daily Dollar/Ko te Tāra o te Rā

Bill Ellwood Memorial Series

This series is a tribute to Bill Ellwood, who wrote much of the Maths Week material from 2006 to 2019. Bill passed away in June 2021.

Set E Day 5

For students



WHAT TO DO FOR STUDENTS

- 1 You may work on your own or with someone else, and your teacher or someone else can help you.
- 2 Answer the questions.
- 3 Each question has a dollar value. Each day's questions total \$100 in value.
- 4 When you have answered the questions, your teacher will give you the answers.
- If you are right, you will get the dollar value for each question. You then you can work out how many dollars you have earned for the day.
- Add the number of dollars you have earned each day in the Daily Dollar questions and get a total for the week. Then you can compare your total for the week with others in your class.
- 7 Perhaps your teacher may award a prize for the highest total for the week!
- 8 Good luck!

SPIRALS

For tasks 1, 2 and 3, you will need a grid to draw on. Your teacher may give you a page with grids already drawn for you to use.

Task 1 - Celtic mazes (\$20)

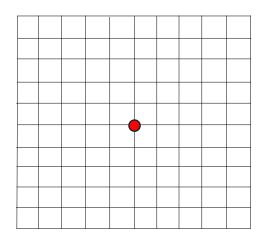
Celtic mazes are straight-line spirals that were drawn all over the world dating back to 3500 BC. They can be described in mathematics using a sequence of numbers.

Start with a grid like the one in the diagram. The positive direction horizontally is to the right and vertically is upwards.

Take the sequence of numbers $1, 2, 3, 4, 5, \ldots$

Start at the centre of the grid.

Follow the instructions below. Each time you move, draw a line for the path that you move on.



Move 1 unit in a positive direction. Turn anticlockwise through 90°.

Move 2 units in a positive direction. Turn anticlockwise through 90°.

Move 3 units in a positive direction. Turn anticlockwise through 90° .

Move 4 units in a positive direction. Turn anticlockwise through 90°.

Move 5 units in a positive direction. Turn anticlockwise through 90°.

Keep going until you get a spiral.

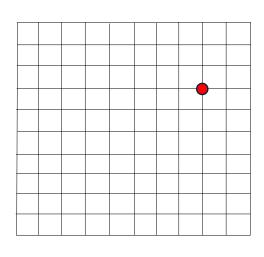
Task 2 (\$20)

Start with a grid. As in Task 1, the positive direction horizontally is to the right and vertically is upwards.

Take the sequence of numbers $1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, \dots$

Start in the grid as shown in the diagram.

Draw a spiral in the same way as you did in Task 1.



Task 3 (\$20)

Draw a spiral using your own sequence. Here are some suggestions.

1, 3, 1, 3, 1, 3, . . .

1, 3, 3, 5, 4, . . .

1, 2, 3, 4, 5, 4, 3, 2, 1 . . .

1, 2, 3, 2, 1, 2, 3, . . .

1, 2, 3, 2, 2, 1, 2, 3, 2, 2, . . .



See if you can make a spiral with at least eight lines that returns to its starting point. Give the sequence of numbers that makes the spiral.

Task 4 - Archimede's spiral (\$20)

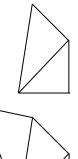
You will need a protractor for this task.

The spiral that you will draw in this question was named after the famous Greek mathematician Archimedes, who lived from 287 BC to 212 BC. He wasn't that keen on the spiral, although it still bears his name.

Start with an isosceles right-angled triangle with the two equallength sides 1 unit in length.

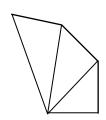
Draw a line of length 1 unit at right angles to the hypotenuse of the triangle. Draw a second triangle as shown.

Draw a line of length 1 unit at right angles to the hypotenuse of the second triangle. Draw a third triangle as shown.



Continue drawing triangles in the same way until you have 15 in total.

Draw a heavy line over the outside edges of the shape. This line is an approximate Archimede's spiral. Instead of just drawing over the lines, you could try drawing a smooth curve to get a better approximation to the spiral.



Task 5 - Equiangular spiral (\$20)

The equiangular spiral often occurs in nature. The pictures on the right show two examples.

The equiangular spiral is also known as a logarithmic spiral or a growth spiral. It is the only type of spiral that does not alter its shape as it grows.



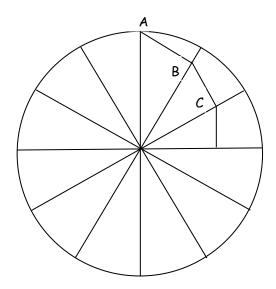


For this task, you will need a compass and a protractor.

Draw a circle with a radius of about 6 cm. In the circle, draw a diameter of the circle that is vertical. Mark the point at the top of the circle where the diameter meets the circle A.

Draw five other diameters that are equally spaced around the circle.

From A, draw a line that meets the next diameter at 90° going clockwise. Mark the point where this line meets the next diameter as B.



From B, draw a line that meets the next diameter at 90°. Label the point where this line meets the next diagonal. Continue doing this until you reach the centre of the circle.

Draw a heavy line over the lines you drew from the points A, B, C, . . . This line is an approximation to an equiangular spiral. Instead of just drawing over the lines, you could try drawing a smooth curve to get a better approximation to the spiral.

See if you can find other some places where equiangular spirals occur.